

Constraints on Modified Gravity from ACT and SPT

Andrea Marchini,¹ Alessandro Melchiorri,¹ Valentina Salvatelli,¹ and Luca Pagano¹

¹*Physics Department and INFN, Università di Roma “La Sapienza”, Ple Aldo Moro 2, 00185, Rome, Italy*

The Atacama Cosmology Telescope (ACT) and the South Pole Telescope (SPT) have recently provided new and precise measurements of the Cosmic Microwave Background anisotropy damping tail. This region of the CMB angular spectra, thanks to the angular distortions produced by gravitational lensing, can probe the growth of matter perturbations and provide a new test for general relativity. Here we make use of the ACT and SPT power spectrum measurements (combined with the recent WMAP9 data) to constrain $f(R)$ gravity theories. Adopting a parametrized approach, we obtain an upper limit on the lengthscale of the theory of $B_0 < 0.86$ at 95% c.l. from ACT, while we get a much stronger limit from SPT with $B_0 < 0.14$ at 95% c.l..

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I. INTRODUCTION

The major goal of modern cosmology is to understand the source of cosmic acceleration. One of the possible solutions to this puzzling phenomenon is to modify general relativity on very large scales in order to allow an accelerating phase in matter-only universes. Examples of such “Modified Gravity” (hereafter MG) models are $f(R)$ theories [1–5] and in the recent years several authors have searched for modified gravity and departures from general relativity in cosmological data [6–21]

The recent precise measurements of the Cosmic Microwave Background damping tail from the Atacama Cosmology Telescope (ACT) [22] and the South Pole Telescope (SPT) [23] are offering a new opportunity to further test MG theories.

The shape of the damping tail of the CMB anisotropies depends strongly from the effect of lensing caused by the intervening matter densities along the line of sight of the CMB photons. CMB lensing therefore probes the growth of perturbations up to redshift $z \sim 6$. Since the amplitude and the evolution of matter perturbations can be drastically altered in MG theories a precise detection of the CMB lensing can place strong constraints on these deviations and possibly identify them (see e.g. [25]).

However the ACT and SPT experiments are reporting quite different constraints on the amount of CMB lensing (see the discussion in [24]). Parametrizing the lensing amplitude by an effective amplitude A_L , that is $A_L = 1$ in case of the standard expected signal and $A_L = 0$ in case of no lensing (see [26] for a definition), the ACT data provide the constraint $A_L = 1.7 \pm 0.38$ at 68% c.l. ([22]), therefore indicating a larger amplitude, while the SPT is more consistent with the standard expectations with $A_L = 0.85 \pm 0.15$, again at 68% c.l. ([23]).

The A_L parameter is clearly an effective parameter and can be used just to indicate possible deviations from the expectations of the standard scenario. It is therefore timely, as we do in this paper, to analyse the results from ACT and SPT in the context of more physically consistent scenarios, as MG theories.

Here we adopt the parametrized modified gravity sce-

nario presented in [40], restricting our analysis to the case of $f(R)$ theories. In this model, the background expansion is identical to the one produced by a cosmological constant, while the evolution of perturbation is altered and depends on a single parameter B_0 that represents the length-scale of the theory [27].

Since the ACT and SPT datasets are providing results that are significantly different, we take a conservative approach to analyse each dataset and discuss the corresponding results separately. The ACT and SPT datasets are combined with the recent data release from nine of observations from the Wilkinson Microwave Anisotropy Probe (WMAP9) [28],

The paper is organized as follows. In Section II we present the modified gravity model considered for our analysis, in Sec. III we describe the analysis method and in Sec. IV we present our results. We conclude in Section V.

II. PARAMETRIZED $f(R)$ GRAVITY

The $f(R)$ theories are currently one of the most popular class of MG models. These models generalize the Einstein-Hilbert action replacing the Ricci scalar with a function of R itself. The generic modified action is

$$S = \int d^4x \sqrt{-g} \left[\frac{f(R)}{2k^2} + \mathcal{L}_m \right] \quad (1)$$

where $k^2 = 8\pi G$ ($c=1$) and \mathcal{L}_m is the matter lagrangian density.

Focusing on this particular MG category is interesting for two main reasons. Firstly, their modified Lagrangian is quite simple and generic, since the modified dynamic at every scale is recovered using only the first order invariant. Secondly, some models belonging to this class have been shown to satisfy both cosmological viability conditions and local tests of gravity, thanks to the chameleon mechanism [37–39].

In order to reproduce the effects of $f(R)$ gravity in the evolution of matter perturbations here we adopt a generic

MG parametrized form, proposed in [40], specializing it to the $f(R)$ case. In this parametrization the background is fixed to that of Λ CDM and the modifications in the linearized Einstein equation are encoded in two scale- and time-dependent parametric function $\mu(k, a)$ and $\gamma(k, a)$

$$k^2\Psi = -\mu(k, a)4\pi G a^2\{\rho\Delta + 3(\rho + P)\sigma\} \quad (2)$$

$$k^2[\Phi - \gamma(k, a)\Psi] = \mu(k, a)12\pi G a^2(\rho + P)\sigma \quad (3)$$

where Ψ and Φ are the two scalar metric potentials in the Newtonian gauge, σ is the anisotropic stress that vanishes for baryons and CDM but not for relativistic species, $\delta \equiv \delta\rho/\rho$ is the density contrast and $\rho\Delta$ is the comoving density perturbation, defined as

$$\rho\Delta = \rho\delta + 3\frac{Ha}{k}(\rho + P)v \quad (4)$$

where v is the velocity field.

It has been shown in [41] that we can recover the $f(R)$ theories choosing the following parametric form for $\mu(k, a)$ and $\gamma(k, a)$

$$\mu(k, a) = \frac{1 + \frac{4}{3}\lambda_1^2 k^2 a^s}{1 + \lambda_1^2 k^2 a^s}, \gamma(k, a) = \frac{1 + \frac{2}{3}\lambda_1^2 k^2 a^s}{1 + \frac{4}{3}\lambda_1^2 k^2 a^s} \quad (5)$$

Viable $f(R)$ models must have $s \sim 4$ in order to closely mimic Λ CDM expansion [27], that is the case we are interested in. Indeed the only free parameter we consider in our analysis is the characteristic lengthscale λ_1 . It is usual expressed in literature in term of the dimensionless parameter B_0 as follows:

$$B_0 = \frac{2H_0^2\lambda_1^2}{c^2} \quad (6)$$

i.e. it gives the lengthscale in units of the horizon scale.

III. DATA ANALYSIS METHOD

Our theoretical models are computed with the public available code MGCAMB [40] v .2 while the analysis is based on a modified version of CosmoMC [29] a Monte Carlo Markov Chain code.

We consider the following set of recent CMB data (publically available on the corresponding web pages): WMAP9 [28], SPT [23], ACT [22] including measurements up to a maximum multipole number of $l_{\max} = 3750$.

For the ACT experiment we use the "lite" version of the likelihood [36] that has been tested to be correct also in the case of the extension respect to Λ -CDM models.

We also consider a gaussian prior on the Hubble constant (hereafter HST prior) $H_0 = 73.8 \pm$

$2.4 \text{ km s}^{-1} \text{ Mpc}^{-1}$, consistently with the measurements of the HST [33].

We include information from measurements of baryonic acoustic oscillations (BAO) from galaxy surveys, combining four datasets: 6dFGRS from [30], SDSS-DR7 from [31], SDSS-DR9 from [34] and WiggleZ from [35]. We refer to this dataset as BAO.

We sample a seven-dimensional set of cosmological parameters, adopting flat priors on them: the B_0 modified gravity parameter, the baryon and cold dark matter densities $\Omega_b h^2$ and $\Omega_c h^2$, the ratio of the sound horizon to the angular diameter distance at decoupling θ , the optical depth to reionization τ , the scalar spectral index n_s , the overall normalization of the spectrum A_s at $k = 0.002 \text{ Mpc}^{-1}$.

Given the tension between the ACT and SPT experiment in the lensing amplitude, we also consider variations in the lensing amplitude parameter A_L as defined in [26]. Finally, the amount of helium abundance in the universe Y_p is fixed by assuming Big Bang Nucleosynthesis in the standard case of three neutrino families.

IV. RESULTS

Our main results are reported in Table I. Since the ACT and SPT datasets are reporting significantly different constraints on B_0 we consider these two datasets separately.

As we can see, both ACT and SPT are not providing any evidence for MG. However, the SPT dataset gives significantly stronger constraints on B_0 ($B_0 < 0.14$ at 95% c.l.) respect to those derived by ACT ($B_0 < 0.90$ at 95% c.l.). The difference appears as even more striking in Figure 1 (Left Panel), where we report the two posteriors on B_0 coming from the two experiments: while SPT strongly constrain B_0 , the posterior from ACT shows a bimodal distribution, suggesting an higher compatibility with modified gravity models. The reason of this difference is mainly due to the different lensing signal present in the ACT e SPT TT spectra (see [24]): since $f(R)$ MG models increase the lensing signal they are more consistent with the larger amplitude of ACT than with the smaller amplitude of SPT. The best fit value for ACT is indeed $B_0 \sim 0.78$ even if this dataset still does not provide any compelling evidence for MG.

The inclusion of the HST prior and of the BAO dataset improves the constraints ($B_0 < 0.12$ at 95% c.l. from SPT and $B_0 < 0.86$ at 95% c.l. from ACT), however not in a significant way, clearly showing that most of the constraining power is coming from the CMB spectrum distortions introduced by gravitational lensing.

It is interesting to consider the impact of MG on the standard cosmological parameters. As we see from the Table, and as already showed in [40], there is little correlation between B_0 and the, standard, six cosmological parameters. We found that the largest correlations are with scalar spectral index n_s and amplitude A_s . How-

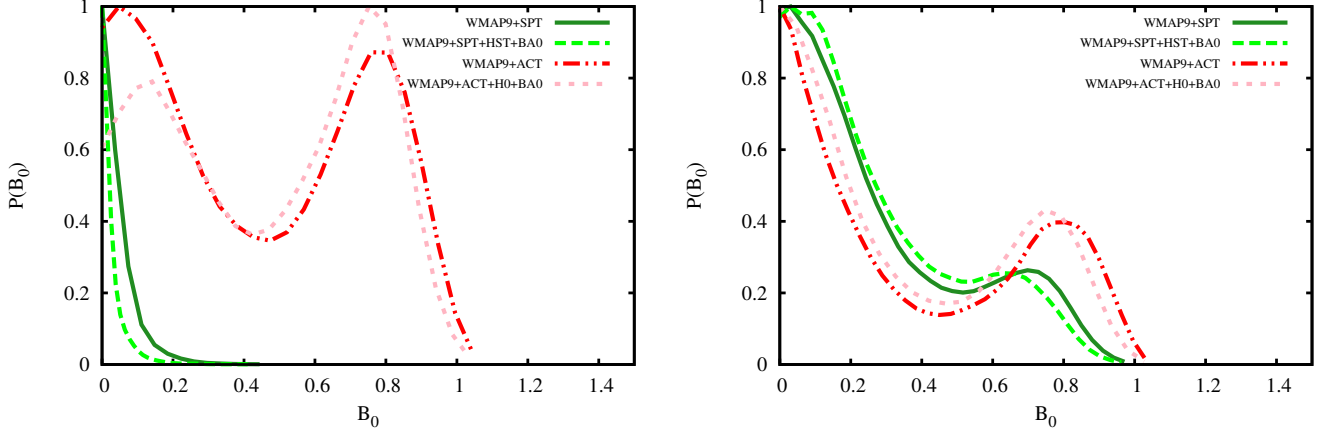


FIG. 1. Posterior distribution functions for the B_0 parameter in the case of $A_L = 1$ (Left panel) and A_L free (Right Panel). The different lensing amplitude measured by ACT makes MG model more consistent with the data and a bimodal posterior distribution is present (Left Panel). When variations in A_L are considered the SPT bound is weaker, while the ACT dataset is more consistent with GR. The bimodal distribution still present in the right panel is due to the low WMAP anisotropy at large angular scales.

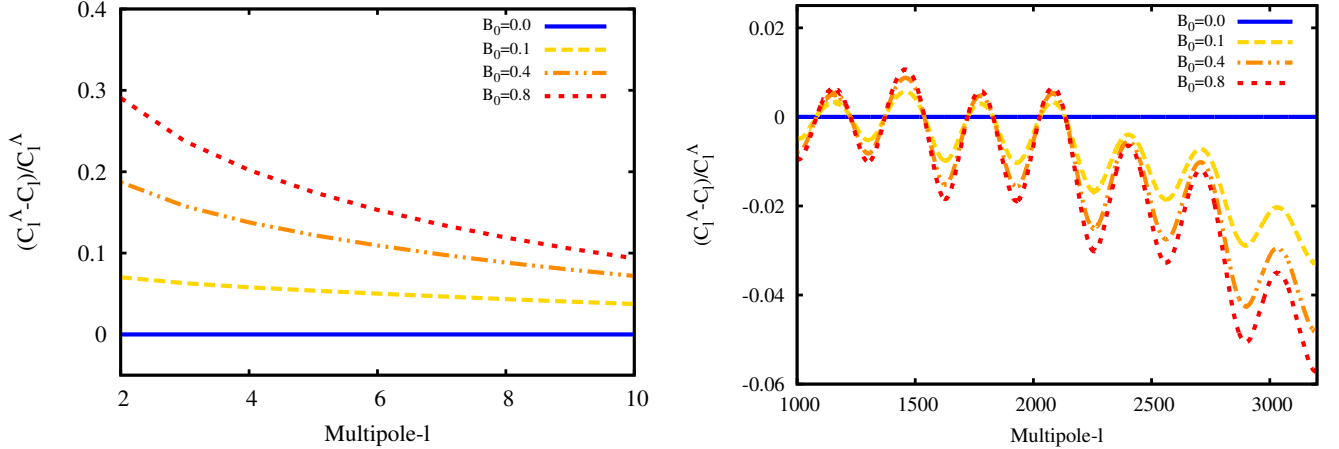


FIG. 2. Effect of B_0 on the CMB angular temperature power spectrum. We plot the differences respect to the standard Λ -CDM model. On the left panel we see that the anisotropy at low multipoles decreases as B_0 is increased. A larger B_0 is therefore more compatible with the low WMAP quadrupole. On the right panel we see that the anisotropies on large angular scales are increased respect to Λ -CDM. The effect is due to an increase in the CMB lensing amplitude.

ever, these correlations changes in function of B_0 . When $B_0 \ll 1$, larger B_0 is in more agreement with smaller n_S and larger A_S . When $B_0 \sim 1$, larger B_0 is in more agreement with larger n_S and smaller A_S .

In order to further test the importance of the lensing signal in constraining modified gravity models we have performed an analysis by letting variations in the lensing amplitude A_L . The results are reported in Table 2. As we can see, the effect of marginalizing over the lensing amplitude is clearly to make weaker the SPT constraint and to leave as unaffected the ACT constraint. However the bimodal distribution present in the ACT case is now suppressed as we can see from Figure 1 (Right Panel) where we plot the posterior distribution functions for B_0 . Moreover, as we can see from the results in Table 2, the

ACT lensing signal is consistent with $A_L = 1$ in the case of MG models. We can therefore conclude that while the ACT data does not show any evidence for MG, the lensing signal is in better agreement with the $A_L = 1$ case in the framework of MG.

It is interesting to note that, while now suppressed from the previous case, the bimodal distribution is still present when A_L varies and it is also now evident in the SPT dataset. The reason is that MG $f(R)$ gravity models produce also a lower quadrupole and lower ℓ temperature anisotropy in agreement with the WMAP data (see Figure 2, left panel).

Parameters	SPT+WMAP9	ACT+WMAP9	SPT+WMAP9+HST+BAO	ACT+WMAP9+HST+BAO
$\Omega_b h^2$	0.02224 ± 0.00034	0.02281 ± 0.00044	0.02235 ± 0.00033	0.02279 ± 0.00040
$\Omega_c h^2$	0.1091 ± 0.0036	0.1142 ± 0.0044	0.1119 ± 0.0023	0.1149 ± 0.0028
100θ	1.0428 ± 0.0010	1.0402 ± 0.0020	1.04257 ± 0.00098	1.0403 ± 0.0019
τ	0.0827 ± 0.013	0.091 ± 0.014	0.080 ± 0.012	0.090 ± 0.013
n_s	0.9676 ± 0.0093	0.973 ± 0.012	0.9633 ± 0.0078	0.9724 ± 0.0096
B_0	< 0.14 (95% c.l.)	< 0.90 (95% c.l.)	< 0.12 (95% c.l.)	< 0.86 (95% c.l.)
H_0 [km/s/Mpc]	72.2 ± 1.7	70.2 ± 2.1	70.9 ± 1.0	70.0 ± 1.3
$\log(10^{10} A_s)$	3.060 ± 0.027	3.174 ± 0.045	3.066 ± 0.025	3.185 ± 0.035
Ω_Λ	0.747 ± 0.018	0.721 ± 0.025	0.733 ± 0.012	0.718 ± 0.015
Ω_m	0.253 ± 0.018	0.279 ± 0.025	0.267 ± 0.0012	0.282 ± 0.015
Age/Gyr	13.689 ± 0.066	13.71 ± 0.10	13.724 ± 0.053	13.714 ± 0.084
D_{3000}^{SZ}	4.2 ± 2.1	—	4.0 ± 2.1	—
D_{3000}^{CL}	4.8 ± 2.0	—	4.8 ± 2.0	—
D_{3000}^{PS}	20.3 ± 2.4	—	20.5 ± 2.3	—
A_{SZ}	—	0.94 ± 0.57	—	0.91 ± 0.56
$\chi^2_{\min}/2$	3808.25	3799.09	3811.41	3800.89

TABLE I. Constraints on the MG parameter B_0 and the standard cosmological parameters described in the text from ACT and SPT combined with WMAP9, HST prior and BAO. We report constraints at 68% confidence level (bounds on B_0 are at 95% c.l.).

Parameters	SPT+WMAP9	ACT+WMAP9	SPT+WMAP9+HST+BAO	ACT+WMAP9+HST+BAO
$\Omega_b h^2$	0.02204 ± 0.00033	0.02294 ± 0.00048	0.02215 ± 0.00032	0.02289 ± 0.00039
$\Omega_c h^2$	0.1154 ± 0.0027	0.1132 ± 0.0043	0.1137 ± 0.0025	0.1144 ± 0.0027
θ	1.04200 ± 0.00097	1.0406 ± 0.0019	1.04233 ± 0.00096	1.0403 ± 0.0018
τ	0.083 ± 0.013	0.090 ± 0.014	0.085 ± 0.013	0.089 ± 0.013
A_L	0.60 ± 0.10	1.25 ± 0.29	0.62 ± 0.11	1.19 ± 0.26
n_s	0.9561 ± 0.0084	0.971 ± 0.011	0.9598 ± 0.0081	0.9699 ± 0.0097
B_0	< 0.73 (95% c.l.)	< 0.91 (95% c.l.)	< 0.77 (95% c.l.)	< 0.85 (95% c.l.)
H_0 [km/s/Mpc]	69.1 ± 1.2	70.8 ± 2.1	70.0 ± 1.1	70.2 ± 1.2
$\log(10^{10} A_s)$	3.083 ± 0.026	3.177 ± 0.040	3.082 ± 0.026	3.183 ± 0.034
Ω_Λ	0.712 ± 0.015	0.727 ± 0.024	0.722 ± 0.013	0.721 ± 0.014
Ω_m	0.288 ± 0.015	0.273 ± 0.024	0.278 ± 0.0013	0.279 ± 0.014
Age/Gyr	13.790 ± 0.055	13.68 ± 0.10	13.760 ± 0.053	13.702 ± 0.079
D_{3000}^{SZ}	5.1 ± 2.3	—	5.0 ± 2.3	—
D_{3000}^{CL}	5.2 ± 2.1	—	5.2 ± 2.1	—
D_{3000}^{PS}	20.0 ± 2.4	—	20.1 ± 2.4	—
A_{SZ}	—	1.9 ± 1.3	—	1.7 ± 1.2
$\chi^2_{\min}/2$	3807.34	3798.95	3808.98	3800.64

TABLE II. Constraints on the $f(R)$ parameter B_0 , the lensing amplitude A_L and the standard cosmological parameters described in the text from ACT and SPT combined with WMAP9, HST prior and BAO. We report constraints at 68% confidence level (bounds on B_0 are at 95% c.l.).

V. CONCLUSIONS

In this brief paper we have presented new constraints on $f(R)$ MG models from the new recent measurements of the CMB damping tail provided by the ACT and SPT experiments. We have found that both experiments show no evidence for deviations from GR. However, while the SPT data significantly improves the previous constraints obtained from similar analysis, the ACT data gives much weaker constraints and shows a bimodal posterior distribution for B_0 . We have attributed this different behaviour to the different amplitude of the lensing signal

detected by those experiments and showed that when the lensing amplitude A_L is let to vary both datasets provide similar constraints. When A_L is varied, we have found that the ACT data does not show any indication for $A_L > 1$ in the framework of MG models. Moreover, a bimodal distribution for B_0 is present in both ACT and SPT datasets when we marginalize over A_L . This is due to the large angular scale regime of the measured CMB spectrum, that prefers a low quadrupole and a bluer spectral index (see e.g. [32]).

Presenting combined results from ACT and SPT spectra as in [44] needs to be done with great care: while

compatible in between two standard deviations in the case of a standard six parameter analysis, the two experiments could show very different constraints in extended theoretical frameworks, especially when the lensing signal plays a significant constraining role.

It is useful to compare the SPT results with previous limits on B_0 present in the literature. The constrain we obtain from WMAP9+SPT+H0+BAO, in the case $A_L = 1$, is much tighter than the constraint of $B_0 < 0.42$ (95% C.L.) obtained from a combined analysis of cosmological data and Integrated Sachs Wolfe data [42] and from the similar constraints $B_0 < 0.4$ (95% C.L.) obtained in [40] and $B_0 < 0.42$ (95% C.L.) from [43], where the datasets considered are slightly different between papers. In [43] they also found a very tight constrain combining with cluster abundance data ($B_0 < 0.001$ 95% C.L.), however this constraint is obtained in the non linear perturbation regime where the simple treatment of $f(R)$ models we adopt here may not be sufficient.

While the SPT provides a much better constraint, one should however also consider it with great caution, given the tension on the lensing amplitude with the ACT

dataset.

Finally, the ACT collaboration has provided a determination of the lensing amplitude also from the four points CMB correlation function (see [22]). This amplitude is perfectly consistent with the standard case, however we prefer here to do not include this dataset for the following conservative reasons: a) we prefer to compare the ACT and SPT datasets at the same power spectrum level; b) the ACT constraint from higher correlations comes from about 50% of the data used in the estimation for the power spectrum (the ACT-E dataset).

The current measurements of CMB lensing will be dramatically improved by the Planck satellite mission, expected to release new data by end on March 2013.

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